

Gyroscopic Inflation

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We propose a new framework for multi-field inflation in which a nearly constant potential energy is maintained during inflation before decreasing rapidly, in a manner analogous to a classical top spinning upright for a long time before falling down. We provide the simplest realization of such dynamics as a well-controlled, weakly-coupled effective field theory with a global shift symmetry. Nonperturbative quantum gravitational effects, which break global symmetries, are suppressed in this model. Primordial gravitational waves may be within experimental reach.

INTRODUCTION AND MODEL

The spectacular data from the cosmic microwave background (CMB) and large-scale structure measurements have firmly established slow-roll inflation [1, 2] as the source of primordial density fluctuations,¹ as well as a solution to the old cosmological conundrums such as the horizon problem [4]. With the ongoing Planck satellite mission and the next generation of CMB experiments, we are entering an exciting era in which we will be probing some of the fundamental parameters of inflationary dynamics.

A challenging problem in slow-roll inflation, from the viewpoint of effective field theory, is how to protect its extremely flat potential against quantum corrections. This problem appears robust, as extreme flatness is required to maintain inflation for a sufficiently long time, while the loop corrections can be ultimately attributed to the need for inflation to end. If inflation were to last forever, we could ensure an exactly flat potential for a slow-roll scalar field, ψ , even in the presence of radiative corrections, by imposing symmetry under the global shift $\psi(x) \rightarrow \psi(x) + c$. But inflation must end, which seems to require violating the shift symmetry to let the potential drop to zero at some point, thereby unleashing the dangerous radiative corrections.

In this letter, we propose a new framework that provides a simple, concrete counterexample to this argument. We dub the framework “gyroscopic inflation,” as its dynamics can be essentially captured by analogy with a symmetric top spinning on a nearly frictionless, horizontal table. The top’s potential energy evolves precisely in the manner required for a successful slow-roll inflation; it stays approximately constant for a long time while the top remains upright and then swiftly drops to zero as the top falls down. Let us highlight two essential features of this dynamics in terms of the Euler angles θ and ψ (the top’s tilt and its spinning), neglecting the slow precession variable ϕ : (i) θ retains a nearly constant potential energy not by slowly rolling over a very flat potential

but by sitting at the origin $\theta = 0$, which remains a stable point for a long time; (ii) The stability of the origin is controlled not by how much the top has turned, ψ , but by how fast it is turning, $\dot{\psi}$, which is invariant under the shift $\psi \rightarrow \psi + c$.

To implement these features in relativistic field theory, we promote ψ to a slow-roll field, and θ to a “waterfall” field, borrowing the terminology of hybrid inflation [5, 6], which shares the feature (i) but differs crucially in regard to (ii); namely, in hybrid inflation, it is ψ itself that controls the stability of θ , so the shift symmetry is necessarily violated.² Now, since θ should be unstable by itself, we write a tachyonic mass-squared term, $+\mu^2\theta^2$, in the lagrangian. In the presence of a sufficiently large $|\dot{\psi}|$, $\theta = 0$ should become a stable point. The leading ψ - θ cross coupling with such a property is $-(\partial_\mu\psi)(\partial^\mu\psi)\theta^2 = -\dot{\psi}^2\theta^2 + (\partial_i\psi)(\partial^i\psi)\theta^2$, which quickly redshifts to $-\dot{\psi}^2\theta^2$ in an expanding spacetime and provides θ with a positive effective mass-squared. Therefore, the simplest model of gyroscopic inflation is given by

$$\begin{aligned} \mathcal{L} = & \frac{1}{2}(\partial\psi)^2 - \frac{m^2}{2}\psi^2 \\ & + \frac{1}{2}(\partial\theta)^2 - \frac{1}{2}\left[-\mu^2 + \frac{(\partial\psi)^2}{\Lambda^2}\right]\theta^2 - \frac{\lambda}{4}\theta^4 \\ & - V_0 + \mathcal{L}_{\text{ct}}, \end{aligned} \quad (1)$$

where m^2 , μ^2 , Λ^2 and λ are all positive real parameters, and V_0 adjusts the energy of the vacua (located at $(\psi, \theta) = (0, \pm\mu/\sqrt{\lambda})$ at tree level) to zero. Internal symmetries imposed on (1) are: (a) two \mathbb{Z}_2 symmetries under which $\theta \rightarrow -\theta$ and $\psi \rightarrow -\psi$, and (b) a global $\psi \rightarrow \psi + c$ shift symmetry broken only by the $m^2\psi^2$ term.³ Being nonrenormalizable, this theory has a physical momen-

¹ See [3] for a recent excellent review.

² Nevertheless, various mechanisms exist to tame the loop corrections in hybrid inflation, e.g., by applying the little higgs mechanism [7, 8] or by making the inflaton and waterfall fields composite [9].

³ This assumption can be made only in the limit of decoupling gravity, $M_{\text{Pl}} \rightarrow \infty$. With gravity turned on, the term $\sqrt{-g}m^2\psi^2$ yields an infinite number of shift-symmetry breaking terms when expanded in terms of the graviton field. The effects of gravity will be discussed below.

tum cutoff proportional to Λ , with the proportionality constant depending on the ultraviolet completion of the theory. \mathcal{L}_{ct} denotes all counterterms that are not only consistent with these symmetries but also *required* by renormalization. In particular, \mathcal{L}_{ct} is exactly invariant under $\psi \rightarrow \psi + c$, since the breaking by $m^2\psi^2$ is soft and does not introduce any ultraviolet divergence that breaks the shift symmetry.⁴

The $m^2\psi^2$ term cannot be omitted. An inflationary phase lasts as long as $|\dot{\psi}|$ remains large enough to keep θ stable at $\theta = 0$. As the universe expands, $|\dot{\psi}|$ decreases via Hubble damping and inflation thus ends eventually, but if m^2 were zero, the damping rate of $\dot{\psi}$ would be of order the expansion rate H itself by dimensional analysis. This would be too fast to give a sufficient number of e-folds. On the other hand, with a nonzero but still small $m^2 \ll H^2$, ψ behaves as a slow-roll field with a damping rate $\sim m^2/H \ll H$, permitting us to obtain enough e-folds.

DYNAMICS OF GYROSCOPIC INFLATION

Let us investigate the dynamics of gyroscopic inflation in its simplest realization (1). We choose an initial condition in which θ is displaced from a vacuum $\langle\theta\rangle$ toward the origin, that is, $\theta^2 \ll \langle\theta\rangle^2$. We also assume a sufficiently large initial $|\dot{\psi}|$ such that the origin $\theta = 0$ is initially stable. The universe will then expand with a rate $H \gtrsim \sqrt{V_0/(3M_{\text{Pl}}^2)}$, quickly redshifting away spatial curvature as well as any spatial gradients of θ and ψ . Therefore, in coordinates in which $ds^2 = dt^2 - [a(t)]^2 dx^i dx^i$, the equation of motions for θ and ψ are given at tree level by

$$\ddot{\theta} + 3H\dot{\theta} + \left(\frac{\dot{\psi}^2}{\Lambda^2} - \mu^2\right)\theta = 0, \quad (2)$$

$$\ddot{\psi} + 3H\dot{\psi} + m^2\psi = 0, \quad (3)$$

where $H \equiv \dot{a}/a$, and $\lambda\theta^3$ is dropped in eq. (2) in accord with the condition $\theta^2 \ll \langle\theta\rangle^2$. Also, in deriving eq. (3), the ψ - θ cross-coupling term in the lagrangian (1) was ignored,⁵ because \mathcal{L}_{ct} is under control as an expansion in powers of θ^2/Λ^2 only if

$$\langle\theta\rangle^2 \ll \Lambda^2, \quad (4)$$

which, together with $\theta^2 \ll \langle\theta\rangle^2$, implies $\theta^2/\Lambda^2 \ll 1$. Finally, the inflation rate H in (2) and (3) is given by

$$H^2 = \frac{V_0}{3M_{\text{Pl}}^2}, \quad (5)$$

provided that the energy density of the θ - ψ system is dominated by V_0 , which we will justify shortly. Now, with H being constant, eq. (2) implies that θ remains stable at $\theta = 0$ as long as $\dot{\psi}^2/\Lambda^2 > \mu_c^2$, where

$$\mu_c^2 \equiv \mu^2 + \left(\frac{3H}{2}\right)^2 \simeq \mu^2. \quad (6)$$

Here, for a swift ending of inflation, we have assumed $\mu^2 \gg (3H/2)^2$ so that θ is not a slow-roll field and will quickly roll down to one of the vacua once it becomes tachyonic. Therefore, at the end of inflation ($t \equiv 0$), we choose $\psi(0) = \pm\mu\Lambda$. Then, since ψ is a slow-roll field (i.e., $m^2 \ll (3H/2)^2$), the solution to eq. (3) is

$$\dot{\psi}(t) = -\frac{m^2}{3H}\psi(t) = \pm\mu\Lambda \exp\left[-\frac{m^2}{3H}t\right]. \quad (7)$$

We can now check the assumption that the energy density of the θ - ψ system is dominated by V_0 . The kinetic and potential energies of θ are well-approximated by 0 and V_0 , respectively, since θ and $\dot{\theta}$ approach zero exponentially at a rate of $\mathcal{O}(H)$, as θ is not a slow-roll field ($\mu^2 \gg H^2$). On the other hand, the kinetic energy of the slowly-rolling ψ is subdominant to its potential energy, $m^2\psi^2/2$. Thus, it suffices to demand $m^2\psi^2/2 \ll V_0$ at an earlier time when the length scale corresponding to the CMB epoch left the horizon, as $|\psi|$ decreases with time. Denoting by N_{CMB} the number of e-folds required to solve the horizon and flatness problems for that length scale, the requirement that $m^2\psi^2/2 \ll V_0$ at $t = -N_{\text{CMB}}/H$ yields the condition

$$\begin{aligned} 1 &\gg \frac{m^2\psi^2}{2V_0} = \frac{1}{V_0} \frac{9H^2\mu^2\Lambda^2}{2m^2} \exp\left[\frac{2m^2}{3H^2}N_{\text{CMB}}\right] \\ &= \frac{3\mu^2\Lambda^2}{2m^2M_{\text{Pl}}^2} \exp\left[\frac{2m^2}{3H^2}N_{\text{CMB}}\right]. \end{aligned} \quad (8)$$

Since $\mu^2 \gg m^2$, this implies

$$\Lambda \ll M_{\text{Pl}}. \quad (9)$$

The seemingly innocuous relation (9) is of fundamental importance. Nonperturbative quantum gravitational effects, such as virtual black hole formation, generically break all global symmetries [11, 12]. Fortunately, (9) ensures that gravity is weakly coupled at the cutoff, thereby exponentially suppressing such effects at the scale where the effective theory is defined, and protecting the global $\psi \rightarrow \psi + c$ symmetry of our model.⁶

⁴ Again, this is true only in the limit of decoupling gravity.

⁵ Therefore, unlike in k -inflation [10], the equation of motion for ψ effectively has a standard kinetic term.

⁶ Chaotic inflation [13] with soft shift symmetry breaking by $m^2\psi^2$ can also be protected from nonperturbative gravitational effects if we lower the cutoff from M_{Pl} to Λ by adding a shift-symmetric, nonrenormalizable interaction $(\partial\psi)^4/\Lambda^4$. Even without lowering the cutoff, it is safe from perturbative gravitational effects for the same reason explained below.

Perturbative graviton loops also violate the shift symmetry, but they can also be fully under control. Recall that we required, in the $M_{\text{Pl}} \rightarrow \infty$ limit, that the $m^2\psi^2$ term be the only shift-symmetry breaking term in (1). We also demanded that \mathcal{L}_{ct} contain only counterterms required by renormalization. Therefore, in perturbation theory, all shift-symmetry violations ultimately stem from $\sqrt{-g}m^2\psi^2$, so that every factor of ψ^2 without a derivative acting on it must be accompanied by a factor of m^2/M_{Pl}^2 . Thus, the condition for graviton loops not to spoil the softly broken shift symmetry is

$$1 \gg \frac{m^2\psi^2}{M_{\text{Pl}}^2\Lambda^2} = \frac{m^2\psi^2}{2V_0} \frac{\Lambda^2}{M_{\text{Pl}}^2} \frac{2V_0}{\Lambda^4}, \quad (10)$$

when the CMB length scale left the horizon. This condition is already satisfied because of (8), (9), and the tree level relation

$$V_0 = \frac{\mu^4}{4\lambda} = \frac{\mu^2\langle\theta\rangle^2}{4} \ll \Lambda^4, \quad (11)$$

which follows from (4) and the obvious requirement that $\mu^2 \ll \Lambda^2$.

The last theoretical consistency check is whether \mathcal{L}_{ct} is under control as an expansion in powers of $(\partial\psi)^2/\Lambda^4$. This amounts to demanding $\dot{\psi}^2/\Lambda^4 \ll 1$ when the CMB length scale left the horizon, which gives

$$\frac{\mu^2}{\Lambda^2} \exp\left[\frac{2m^2}{3H^2}N_{\text{CMB}}\right] \ll 1. \quad (12)$$

Note that this is stronger than the condition $\mu^2 \ll \Lambda^2$.

PHENOMENOLOGY

We now study the phenomenology of the simplest model (1). Since primordial perturbations are only generated by degrees of freedom lighter than H , the dimensionless scalar power spectrum $\Delta_{\mathcal{R}}^2$ does not depend on the heavy field θ . Therefore, just as in single-field slow-roll inflation, we have

$$\begin{aligned} \Delta_{\mathcal{R}}^2|_{\text{CMB}} &= \left(\frac{H^2}{2\pi\dot{\psi}}\right)^2 \Big|_{\text{CMB}} \\ &= \frac{H^4}{4\pi^2\mu^2\Lambda^2} \exp\left[-\frac{2m^2}{3H^2}N_{\text{CMB}}\right], \end{aligned} \quad (13)$$

for which the 7-year WMAP data [14] gives

$$\Delta_{\mathcal{R}}^2|_{\text{CMB}} = (2.43 \pm 0.11) \times 10^{-9}. \quad (14)$$

Similarly, the tilt of the scalar power spectrum, $n_s - 1$, can be computed as in single-field inflation by $n_s - 1 = (2\eta - 6\varepsilon)|_{\text{CMB}}$, where the slow-roll parameters for our model are given by

$$\varepsilon = \frac{1}{2} \left(\frac{M_{\text{Pl}} m^2 \psi}{V_0} \right)^2, \quad \eta = \frac{M_{\text{Pl}}^2 m^2}{V_0}. \quad (15)$$

Since η is constant and $\varepsilon/\eta = (m^2\psi^2/2)/V_0 \ll 1$ due to (8), we obtain

$$n_s - 1 \simeq 2\eta = \frac{2m^2}{3H^2}. \quad (16)$$

This should be compared to the current constraint [14], $n_s - 1 = -0.032 \pm 0.012$ (68% CL). To be within 3σ of the central value, let us impose $\eta < 2 \times 10^{-3}$. This then ensures that a sufficient number of e-folds can easily be accommodated, since the exponential factor in (8) and (12) is $\simeq 1$ for N_{CMB} of $\mathcal{O}(10^2)$.

To see how all the theoretical consistency conditions are satisfied, we express them in terms of μ^2 , Λ^2 , η , $\Delta_{\mathcal{R}}^2$, and r . To eliminate the dependence of our expressions on N_{CMB} , m^2 , and H^2 , we use the relations (13), (16), and

$$r = 16\varepsilon|_{\text{CMB}} = \frac{2}{\pi^2\Delta_{\mathcal{R}}^2} \frac{H^2}{M_{\text{Pl}}^2}, \quad (17)$$

respectively. The condition (8) implies an upper bound on the tensor-to-scalar ratio r :

$$r \ll 16\eta < 3.2 \times 10^{-2}. \quad (18)$$

The current experimental bound is $r < 0.20$ at 95% CL [14]. On the other hand, the condition (4) becomes

$$\begin{aligned} 1 &\gg 6\pi^2\Delta_{\mathcal{R}}^2 r \frac{M_{\text{Pl}}^4}{\mu^2\Lambda^2} \\ &\simeq \frac{\Delta_{\mathcal{R}}^2}{2.43 \times 10^{-9}} \frac{r}{3.2 \times 10^{-2}} \left(\frac{0.03}{\mu/\Lambda}\right)^2 \left(\frac{0.05}{\Lambda/M_{\text{Pl}}}\right)^4. \end{aligned} \quad (19)$$

Combined with the bound on r from (18), this implies that the condition (4) can be satisfied with large separations among μ , Λ , and M_{Pl} , thus keeping the effective theory under control as well as suppressing nonperturbative gravitational effects.

The rest of the consistency conditions are easy to satisfy. The condition (12) becomes

$$\begin{aligned} 1 &\gg \frac{\pi^2\Delta_{\mathcal{R}}^2 r^2}{16} \frac{M_{\text{Pl}}^4}{\Lambda^4} \\ &= 2 \times 10^{-7} \frac{\Delta_{\mathcal{R}}^2}{2.43 \times 10^{-9}} \left(\frac{r}{3.2 \times 10^{-2}}\right)^2 \left(\frac{0.05}{\Lambda/M_{\text{Pl}}}\right)^4, \end{aligned} \quad (20)$$

ensuring that the derivative expansion in $(\partial\psi)^2$ is well under control. The slow-roll condition $m^2 \ll (3H/2)^2$ is automatically satisfied, as $m^2/(3H/2)^2 = 4\eta/3 \ll 1$. Finally, the condition $\mu^2 \gg (3H/2)^2$ becomes

$$\begin{aligned} 1 &\gg \frac{9\pi^2\Delta_{\mathcal{R}}^2 r \Lambda^2}{8} \frac{M_{\text{Pl}}^2}{\mu^2 \Lambda^2} \\ &= 4 \times 10^{-4} \frac{\Delta_{\mathcal{R}}^2}{2.43 \times 10^{-9}} \frac{r}{3.2 \times 10^{-2}} \left(\frac{0.03}{\mu/\Lambda}\right)^2 \left(\frac{0.05}{\Lambda/M_{\text{Pl}}}\right)^2, \end{aligned} \quad (21)$$

which is robustly satisfied.

DISCUSSION AND OUTLOOK

We have proposed a new framework, gyroscopic inflation, in which the stability of the field driving the expansion is controlled by a slow-roll field with a shift symmetry that is only softly broken. We have formulated the simplest realization and examined the conditions for it to be a consistent effective field theory. We have seen that those consistency conditions, combined with observational constraints, lead to the prediction that the tensor-to-scalar ratio for primordial perturbations should be $r \ll 3 \times 10^{-2}$. Detection of primordial gravitational waves may therefore be within reach of the next generation of CMB experiments, $r \gtrsim \mathcal{O}(10^{-2})$.

On the other hand, the simplest consistent effective field theory of inflation with a softly broken shift symmetry is $m^2\psi^2$ chaotic inflation [13], which predicts $r = 4(1 - n_s) = 0.13$ for the current central value of $1 - n_s = 0.032$. It is therefore possible that this model could be ruled out by future CMB experiments. Then, divorcing r from $1 - n_s$ would require introducing a second field that dominates the potential energy. Preserving the shift symmetry would then lead to gyroscopic inflation.

Like hybrid inflation [6], the minimal gyroscopic model has a tension with the measurement of $n_s - 1$, although not so severe as to exclude it. Improving this would require an additional degree of freedom with mass $\ll H$. The model-building challenge would be to incorporate such an extension as an integral part of gyroscopic inflation, e.g., by embedding ψ and the new field into a single multiplet.

In both the minimal and improved models, non-Gaussianity in primordial perturbations deserves further investigation. In gyroscopic inflation, in contrast to conventional slow-roll inflation, a nearly constant potential does not necessarily imply that the slow-roll field ψ is almost free, as it has derivative interactions. Therefore, non-Gaussianity should be present at some level even in the simplest model, and could possibly be more pronounced in the improved models. Furthermore, in an improved model, the existence of additional light degrees of freedom would generically lead to non-adiabaticity in the primordial perturbations.

Another reason to modify the simplest model relates to the issue of ultraviolet (UV) completion of the effective theory (1). One might think that the softly broken shift symmetry for ψ should indicate that ψ is the pseudo-Nambu-Goldstone boson (pNGB) of a broken symmetry. The problem is that the value of ψ during inflation is larger than Λ , and hence larger than its

decay constant. For example, at the end of inflation we have $|\psi|/\Lambda = 3H|\dot{\psi}|/(m^2\Lambda) = 3H\mu/m^2 \gg 1$. As in $m^2\psi^2$ chaotic inflation, this does not invalidate the *effective* theory, since the shift symmetry is only softly broken and the shift-symmetry breaking graviton loops are negligible, as we checked in (10). However, a pNGB field value much larger than its decay constant is possible only if the broken symmetry group is non-compact (e.g., \mathbb{R} rather than $U(1)$). This is believed to be incompatible with quantum gravity [12]. One possible resolution may be that a large field value effectively arises as a collective effect, as in N -flation [15]. Another possibility is that the noncompactness of ψ could be related to flat directions in a supersymmetric theory.

To summarize, both the measurement of $n_s - 1$ and the consistency of the UV completion motivate introducing additional light degrees of freedom. Non-Gaussianity and non-adiabaticity may then probe how the minimal model of gyroscopic inflation is extended.

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